Problema 9.3.5.

Simplificaţi următoarele funcţii booleene de patru variabile date prin valorile de 1, utilizând metoda lui Quine:

1. *f*5(1,1,1,1)=1, *f*5(0,1,0,1)=1, *f*5(0,1,1,1)=1, *f*5(1,1,1,0)=1, *f*5(1,1,0,0)=1, *f*5(1,0,0,0)=1, *f*5(1,0,0,1)=1, *f*5(0,0,0,1)=1;

S*f* ={(0,0,0,1),(1,0,0,0), (0,1,0,1),(1,0,0,1),(1,1,0,0), (0,1,1,1),(1,1,1,0), (1,1,1,1)}

Tabelul de factorizare:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Grupul |  | *x*1 | *x*2 | *x*3 | *x*4 |  |
| *I* | *√* | 0 | 0 | 0 | 1 | *m*1 |
|  | *√* | 1 | 0 | 0 | 0 | *m*8 |
| *II* | *√* | 0 | 1 | 0 | 1 | *m*5 |
|  | *√* | 1 | 0 | 0 | 1 | *m*9 |
| *√* | 1 | 1 | 0 | 0 | *m*12 |
| *III* | *√* | 0 | 1 | 1 | 1 | *m*7 |
| *√* | 1 | 1 | 1 | 0 | *m*14 |
| *IV*  *Factorizare simplă* | *√* | 1 | 1 | 1 | 1 | *m*15 |
|  |  | 0 | - | 0 | 1 | *m*1∨ *m*5 = *max*1=¯ *x*1¯*x*3*x*4 |
| *V*=*I*+*II* |  | - | 0 | 0 | 1 | *m*1∨ *m*9= *max*2=¯ *x*2¯*x*3*x*4 |
|  |  | 1 | 0 | 0 | - | *m*8∨ *m*9= *max*3= *x*1¯ *x*2¯*x*3 |
|  |  | 1 | - | 0 | 0 | *m*8∨ *m*12= *max*4= *x*1¯ *x*3¯*x*4 |
| *VI*=*II*+*III* |  | 1 | 1 | - | 0 | *m*12∨ *m*14= *max*5= *x*1*x*2¯*x*4 |
|  | 0 | 1 | - | 1 | *m*5∨ *m*7= *max*6=¯*x*1*x*2*x*4 |
| *VII*=*III*+*IV* |  | 1 | 1 | 1 | - | *m*14∨ *m*15= *max*7= *x*1*x*2*x*3 |
|  | - | 1 | 1 | 1 | *m*7∨ *m*15= *max*8= *x*2*x*3*x*4 |

*Nu avem Factorizare dublă*

M( *f* )={ *max*1, *max*2, *max*3, *max*4, *max*5, *max*6, *max*7, *max*8 }

Tabelul de identificare a monoamelor centrale:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| mon.  max.  mintermi | *max*1 | *max*2 | *max*3 | *max*4 | *max*5 | *max*6 | *max*7 | *max*8 |
| *m*1 | \* | \* |  |  |  |  |  |  |
| *m*8 |  |  | \* | \* |  |  |  |  |
| *m*5 | \* |  |  |  |  | \* |  |  |
| *m*9 |  | \* | \* |  |  |  |  |  |
| *m*12 |  |  |  | \* | \* |  |  |  |
| *m*7 |  |  |  |  |  | \* |  | \* |
| *m*14 |  |  |  |  | \* |  | \* |  |
| *m*15 |  |  |  |  |  |  | \* | \* |

C( *f* )=∅

M( *f* )≠ C( *f* ), C( *f* ) =∅ ⇒ cazul III al algoritmului de simplificare

Determinarea formelor simplificate (backtracking/greedy):

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| mon.  max.  mintermi | *max*1 | *max*2 | *max*3 | *max*4 | *max*5 | *max*6 | *max*7 | *max*8 |
| *m*1 | \* | \* |  |  |  |  |  |  |
| *m*8 |  |  | \* | \* |  |  |  |  |
| *m*5 | \* |  |  |  |  | \* |  |  |
| *m*9 |  | \* | \* |  |  |  |  |  |
| *m*12 |  |  |  | \* | \* |  |  |  |
| *m*7 |  |  |  |  |  | \* |  | \* |
| *m*14 |  |  |  |  | \* |  | \* |  |
| *m*15 |  |  |  |  |  |  | \* | \* |

*f*5’(1)(*x*1, *x*2, *x*3, *x*4)= *h*1(*x*1, *x*2, *x*3, *x*4)= *max*1∨ *max*3∨ *max*5∨ *max*8=¯ *x*1¯*x*3*x*4∨ *x*1¯ *x*2¯*x*3∨ *x*1*x*2¯*x*4∨ *x*2*x*3*x*4

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| mon.  max.  mintermi | *max*1 | *max*2 | *max*3 | *max*4 | *max*5 | *max*6 | *max*7 | *max*8 |
| *m*1 | \* | \* |  |  |  |  |  |  |
| *m*8 |  |  | \* | \* |  |  |  |  |
| *m*5 | \* |  |  |  |  | \* |  |  |
| *m*9 |  | \* | \* |  |  |  |  |  |
| *m*12 |  |  |  | \* | \* |  |  |  |
| *m*7 |  |  |  |  |  | \* |  | \* |
| *m*14 |  |  |  |  | \* |  | \* |  |
| *m*15 |  |  |  |  |  |  | \* | \* |

*f*5’(2)(*x*1, *x*2, *x*3, *x*4)= *h*2(*x*1, *x*2, *x*3, *x*4)= *max*2∨ *max*4∨ *max*6∨ *max*7=¯ *x*2¯*x*3*x*4∨ *x*1¯ *x*3¯*x*4∨¯*x*1*x*2*x*4∨ *x*1*x*2*x*3

Problema 9.3.6.

Simplificaţi următoarele funcţii booleene de trei variabile date prin zerourile acestora, utilizând metoda lui Quine:

1. *f*5(0,0,0) = *f*5(1,1,0) = *f*5(1,1,1) = 0;

S*f* ={(0,0,1),(0,1,0),(0,1,1),(1,0,0),(1,0,1)} ={(0,0,1),(0,1,0),(1,0,0), (0,1,1),(1,0,1)}

Tabelul de factorizare:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Grupul |  | *x*1 | *x*2 | *x*3 |  |
| *I* | *√* | 0 | 0 | 1 | *m*1 |
|  | *√* | 0 | 1 | 0 | *m*2 |
|  | *√* | 1 | 0 | 0 | *m*4 |
| *II*  *Factorizare simplă* | *√* | 0 | 1 | 1 | *m*3 |
| *√* | 1 | 0 | 1 | *m*5 |
| *III=I+II* |  | 0 | - | 1 | *m*1∨ *m*3= *max*1=¯ *x*1*x*3 |
|  | 0 | 1 | - | *m*2∨ *m*3= *max*2=¯ *x*1*x*2 |
|  |  | 1 | 0 | - | *m*4∨ *m*5= *max*3=*x*1¯*x*2 |
| *Nu avem Factorizare dublă* |  | - | 0 | 1 | *m*1∨ *m*5= *max*4=¯ *x*2*x*3 |

M( *f* )={ *max*1, *max*2, *max*3, *max*4}

Tabelul de identificare a monoamelor centrale:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| mon.  max.  mintermi | *max*1 | *max*2 | *max*3 | *max*4 |
| *m*1 | \* |  |  | \* |
| *m*2 |  | \* |  |  |
| *m*4 |  |  | \* |  |
| *m*3 | \* | \* |  |  |
| *m*5 |  |  | \* | \* |

C( *f* )={ *max*2, *max*3}

M( *f* )≠ C( *f* ), C( *f* ) ≠∅ ⇒ cazul II al algoritmului de simplificare

*g*(*x*1, *x*2, *x*3)= *max*2∨ *max*3 =¯ *x*1*x*2∨ *x*1¯*x*2

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| mon.  max.  mintermi | *max*1 | *max*2 | *max*3 | *max*4 |
| *m*1 | \* |  |  | \* |
| *m*2 |  | \* |  |  |
| *m*4 |  |  | \* |  |
| *m*3 | \* | \* |  |  |
| *m*5 |  |  | \* | \* |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| mon.  max.  mintermi | *max*1 | *max*2 | *max*3 | *max*4 |
| *m*1 | \* |  |  | \* |
| *m*2 |  | \* |  |  |
| *m*4 |  |  | \* |  |
| *m*3 | \* | \* |  |  |
| *m*5 |  |  | \* | \* |

*h*1(*x*1, *x*2, *x*3)= *max*1

*h*2(*x*1, *x*2, *x*3)= *max*4

*f*5’(1)(*x*1, *x*2, *x*3)= *g*(*x*1, *x*2, *x*3) ∨ *h*1(*x*1, *x*2, *x*3)= *max*2∨ *max*3∨ *max*1 =¯ *x*1*x*2∨ *x*1¯*x*2∨¯ *x*1*x*3

*f*5’(2)(*x*1, *x*2, *x*3)= *g*(*x*1, *x*2, *x*3) ∨ *h*2(*x*1, *x*2, *x*3)= *max*2∨ *max*3 ∨ *max*4 =¯ *x*1*x*2∨ *x*1¯*x*2∨¯ *x*2*x*3